Unsteady Heat Convection Over Circular Cylinders

Numerical calculations of the velocity and temperature fields were conducted for unsteady flows over a single circular cylinder and a bundle of cylinders. Periodic disturbances, natural or externally imposed, were considered. Periodic and averaged heat transfer rates were examined. Results compare favorably with experimental data. Calculations were carried out to predict heat transfer rates in regimes of unsteady flows that have not been tested experimentally.

B. K. Kim, D. J. VandenBrink, M. S. Cramer, D. P. Telionis
Department of Engineering Science and Mechanics
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061

Introduction

The flow and heat transfer over circular cylinders has been adopted by many investigators as a test case for analytical and experimental studies. Moreover, single or multiple cylinders are very often engineering elements in a variety of important applications, as for example in heat exchangers. The study of this problem therefore provides an understanding of basic characteristics of the flow and heat transfer, readily available for comparison with a rich collection of earlier investigations, as well as useful practical guidelines for the design engineer.

The flow and heat transfer over groups of bodies such as a bundle of cylinders in a heat exchanger is a formidable problem to solve at high Reynolds numbers. This is because it involves a wide spectrum of length scales and therefore requires a very complex computational grid. In this paper we demonstrate how a very old technique, free streamline theory, can be coupled with modern computational methods such as finite-difference and finite-element methods to yield realistic and practical solutions. Moreover, we extend the method to study numerically for the first time the effect of natural vortex shedding on the mean and unsteady heat transfer. Alternative methods based on full Navier-Stokes equations (see extensive reviews in Telionis, 1981) are very costly and encounter difficulties of convergence at high Reynolds numbers. Moreover, even if a converged solution is at hand, the results may be physically inaccurate if an embedded free shear layer—or loosely speaking the boundaries of the wake—are not correctly captured. The method proposed is substantially less expensive to run, provides good physical insight, and may reliably predict the outline of open or closed wakes for large Reynolds numbers.

In most practical situations flow is unsteady either due to natural hydrodynamic oscillations or because of mechanical disturbances. Examples of these two types of disturbances are vortex shedding and mechanical vibrations, respectively. The problem of vortex shedding over a circular cylinder has been the topic of investigation of a very large number of experimental efforts, originating with Strouhal (1878). Reviews on the topic and compilations of data can be found in McCroskey (1977), Savkar (1976), and Sarpkaya (1979). It is remarkable that the Strouhal number, the reduced frequency of shedding, is almost constant and approximately equal to 0.21 for subcritical Reynolds numbers and to 0.30 for supercritical Reynolds numbers. For cascades of circular cylinders the Strouhal number depends on the array configuration and the distances between the cylinders (Savkar, 1976).

Heat transfer over circular cylinders has been studied experimentally by many investigators, as described in classical texts (Eckert and Drake, 1972) and review articles (Zukauskas, 1972; Morgan, 1975). Unsteady heat transfer effects have also been considered, as for example the effects of freestream turbulence (Morgan 1975; Lowery and Vachon 1975; Yardi and Sukhatman 1978; Sunden 1979), and the effects of sound waves (Hribar 1972; Repin 1978). Closer to the present investigation are studies of heat transfer responses to mechanical oscillations of the model (Sreenivasan and Ramachandran, 1961; Saxena and Laird, 1978). A large number of approximate calculations appeared in the 1950's and early 1960's, as reviewed and evaluated by Spalding and Pun (1962). However, to the knowledge of the present authors, no numerical calculations of unsteady heat transfer over circular cylinders have appeared so far.

The heat transfer gauges available today have a very limited frequency range. As a result, no experimental data on unsteady heat transfer are available. Initial efforts of the present group (Campbell and Diller, 1985) indicate that periodic changes in the flow may generate nonlinear contributions to heat transfer.

The present address of B. K. Kim is Department of Chemical Engineering, University of Ulsan, Ulsan, Korea 690.
The present address of D. J. VandenBrink is Department of Mechanical Engineering, Western Michigan University, Kalamazoo, MI 49008.

Our earlier analytical studies (Telionis and Romaniuk, 1977) were based on perturbations about a small amplitude of oscillation. In the present study we integrate numerically the boundary layer equations in a three-coordinate space for amplitudes as large as 15% of the free stream. The outer flow and the wake shape are calculated by a finite-element method that is allowed to interact with the finite-difference boundary-layer solution and a constant-pressure wake. Our heat transfer calculations terminate at separation. Investigations in the separated region will be reported in a later paper.

Boundary Layer

For two-dimensional incompressible attached flow, the governing boundary-layer equations in dimensionless form are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U_e}{\partial t} + U_e\frac{\partial U_e}{\partial x} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{PrRe}\frac{\partial^2 T}{dy^2} + \frac{Ec}{Re}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

The freestream velocity, U_∞, and temperature, T_∞, the length of the model, L, and the quantity, L/U_∞, are used here to generate dimensionless quantities. Conditions are imposed at the wall and the edge of the boundary layer for the temperature and the velocity field, and the flow is assumed to be periodic. The boundary layers are driven by the outer flow, steady or unsteady. The boundary conditions imposed on Eqs. 1–3 are

$$u = v = 0, \quad T = T_w \quad \text{at } y = 0 \tag{4}$$

$$u \rightarrow U_e, \quad T \rightarrow T_e \quad \text{at } y \rightarrow \Delta \tag{5}$$

where unsteadiness is therefore introduced into the boundary layer from the outer flow.

A computer code has been developed earlier for the numerical solution of Eqs. 1 and 2 by a finite-difference method (Telionis et al., 1973). The energy equation has now been added to the program. The field is swept first in the downstream direction, then time is incremented and the process is repeated. An upwind differencing scheme (Telionis et al., 1973; Telionis, 1981) has been included to account for reversing flows. This is necessary to conform with the Courant-Friedrichs-Lewy criterion and guarantee stability and convergence of the numerical scheme (Telionis, 1981).

The upwind scheme employed here is a zigzag scheme interweaving x-differences of the present, t, and the past, $t - \Delta t$, time planes. This is therefore a *second-order* scheme, which inevitably introduces some numerical damping in the solution. This however is innocuous in boundary layer solutions, since the second derivative, $\partial^2 u/\partial x^2$, is missing from the equation. The authors' group has compared zigzag and characteristic upwind schemes and found the first equally effective and accurate for the present problem. This method allows us to proceed beyond the point of zero skin friction and terminate the boundary layer calculation at the instantaneous position of separation (Telionis, 1981; Telionis and Tsahalis, 1974; Sears and Telionis, 1975).

In the present study we investigate periodic fields. These are the fields due to vortex shedding over circular cylinders. The process can be either natural or forced. Arbitrary initial conditions are assumed and the calculations are allowed to proceed through a few oscillations until a periodic flow is established. It is important to note that the disturbance of the vortex shedding process travels instantly upstream through the potential flow. The edge conditions for the boundary layers are therefore oscillating at all points in phase. There are no traveling waves. Phase differences are generated only across the boundary layers.

Outer Flow

Laplace's equation is solved in the outer flow by the method of finite elements. The solution is essentially based on minimization of the functional

$$I = -\frac{1}{2}\iint (\nabla\phi \cdot \nabla\phi)ds + \int \phi u_n d\ell \tag{6}$$

where the surface and line integrals extend over the domain of interest and its periphery, respectively. With three-node, triangular finite elements, ϕ is approximated within the elements by polynomials. This method is well established and well documented in literature (Martin and Carey, 1973). The novel element introduced here is actually a very old technique for modeling wakes: the idea of a free streamline. For a good account of references, dating back to Kirchoff's original contribution in the past century, the reader is referred to Kiya et al. (1977). The classical condition requires that on the line separating the free flow from the wake the pressure be constant.

In the present case an arbitrary wake shape is assumed and the solution is relaxed through a number of iterations until the proper condition is met. It is important to note that unsteady inviscid flow is governed by the same equation as steady flow. However, here nonlinearities enter through the modeling of a "breathing" wake and the nonlinearity of the boundary condition. The pressure deviates from its quasi-steady distribution. It is calculated according to Bernoulli's equation

$$\frac{\partial\phi}{\partial t} + p/\rho + \frac{(\nabla\phi)^2}{2} = f(t)$$

This in turn greatly influences the iteration process since the free streamline condition is based on the local value of pressure. An example of an initial assumption of the wake shape and the computational grid is shown in Figure 1. The grid is flexible to allow for readjustment of the free streamline shape, until the condition of constant pressure is met. Results calculated in this way predict an open wake for a single column of circular cylinders, Figure 2, and a closed wake for a circular cylinder in a bundle, Figure 3. In both cases, shear-free symmetry planes are the conditions representing the effect of the infinite column of cylinders. In both cases presented in these figures the point of separation was assumed at $\theta = 90°$. These were purely inviscid calculations. Interacting viscous-inviscid solutions and relaxation of the separation assumption are reported in later sections.

Stagnation Flow

A serious difficulty is encountered in the boundary layer calculations, in case the point of stagnation is not fixed on the body. This is exactly the case here. Earlier studies as well as our pre-

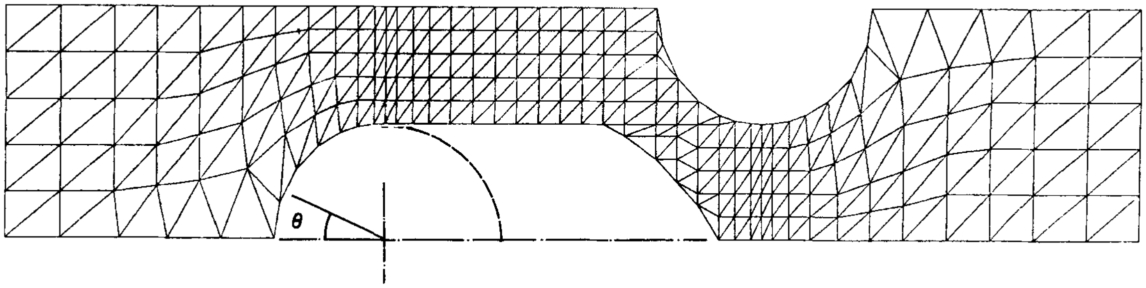

Figure 1. Finite elements for solution of the problem.
Boundary of wake of first cylinder is assumed in the crude form shown, to be relaxed later by iteration.

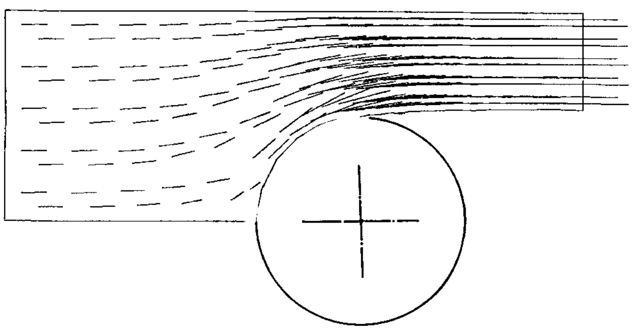

Figure 2. Velocity field and shape of the wake of a circular cylinder confined between two flat plates.
Inviscid results assuming that flow separates at $\theta = 90°$.

liminary experimental results indicate that the point of stagnation performs oscillatory excursions over the circular cylinder with amplitudes of approximately 3°. The problem is not generated by a poor choice of variables. There is a deeper physical reason. The outer flow fluctuates in direction, generating velocity fluctuations near the wall which may be in the opposite sense. As a result, one cannot march in the direction of positive ξ or x without encountering negative flow.

This difficulty was overcome by matching the numerical calculations with an asymptotic solution of the full Navier-Stokes equation for the problem of oscillating stagnation flow. This is essentially an inner expansion, valid for small values of the distance from the origin and small ratios of boundary layer thickness to the radius of the cylinder.

The Navier-Stokes equations in their dimensionless form read

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{8}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{9}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{RePr}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{10}$$

Consider a flow impinging on a flat wall $y = 0$. If the flow far from the wall contains a periodic disturbance in the velocity component parallel to the wall, then the boundary conditions read

$$u = ax + b\cos\omega t + o(1) \tag{11}$$

$$v = -ay + o(y) \tag{12}$$

$$p = b[\omega b\sin\omega t - a\cos\omega t] - \frac{a^2}{2}(y^2 + x^2) + H(t) + o(1) \tag{13}$$

$$T = T_\infty \tag{14}$$

as $y^2 + x^2 \rightarrow \infty$. On the wall, we impose the standard condition

$$u = v = 0, \quad \text{at} \quad y = 0 \tag{15}$$

A solution was found in the following form,

$$v = -f(y) \tag{16}$$

$$u = xf'(y) + g(y)\cos\omega t + h(y)\sin\omega t \tag{17}$$

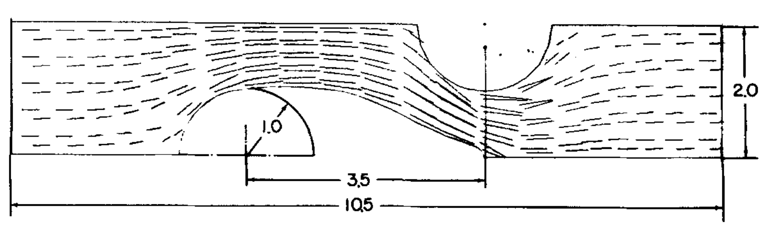

Figure 3. Velocity field and shape of wake of a circular cylinder in a bundle.
Distances rendered dimensionless by radius of cylinders. Inviscid solution assuming separation at $\theta = 90°$.

$$p = m(t) - \frac{1}{Re} f' - \frac{f^2}{2} - \frac{x^2 a^2}{2} + xb(\omega \sin \omega t - a \cos \omega t) \quad (18)$$

$$T = T(y) \quad (19)$$

where the functions f, g, h, and T satisfy the following ordinary differential equations and boundary conditions

$$\frac{1}{Re} f''' + ff'' - f'^2 + a^2 = 0 \quad (20)$$

$$f'(\infty) \rightarrow a, \quad f(0) = f'(0) = 0 \quad (21)$$

$$\frac{1}{Re} g'' + fg' - fg' - \omega h + ab = 0 \quad (22)$$

$$\frac{1}{Re} h'' + fh' - f'h + \omega g - \omega b = 0 \quad (23)$$

$$g(\infty) \rightarrow b, \quad h(\infty) \rightarrow 0, \quad g(0) = h(0) = 0 \quad (24)$$

$$T'' + f\,Pr Re\, T' = 0 \quad (25)$$

$$T(0) = T_W, \quad T(\infty) \rightarrow T_\infty \quad (26)$$

This system of equations has been solved numerically by a standard collocation scheme. Typical results showing the dimensionless counterparts of the functions f, g, and h are displayed in Figure 4. Solutions of the above equations and plots of the functions f, g, and h for a wide range of the parameters will be found in Kim (1984) along with more details about the method and more results.

This solution has been incorporated in our computer code to provide the initial profiles for the unsteady calculation. The method has then been used to generate the instantaneous velocity and temperature field over the attached region of the boundary layer.

Method of Solution

The boundary-layer equations were solved numerically by a finite-difference method. Our computational method sets up a

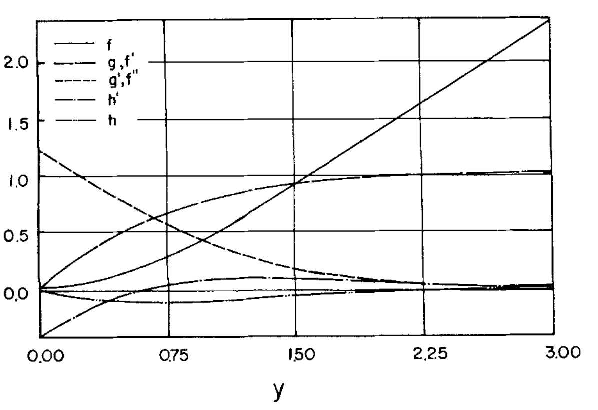

Figure 4. Dimensionless functions defined by Eqs. 16–19 for $\omega = 0$.

three-dimensional grid with variable mesh sizes along two spatial directions, x, y, and time t. Integration then proceeds by marching first along the x direction and then along the t direction. Initial conditions for the $t = 0$ plane are generated by calculating first the steady flow that corresponds to $U_e(x, 0)$. Initial conditions at the $x = 0$ plane are provided by the asymptotic solution described in the previous section. An implicit method of calculation was employed as described in earlier publications (Telionis et al. 1973; Telionis and Tsahalis 1974; Telionis 1981).

Laplace's equation was solved in the outer flow and the wake by finite elements as described in the section above on outer flow. Viscous-inviscid interaction was achieved by allowing a constant-pressure wake to develop downstream of the point of separation. More specifically, the outer code was allowed to relax, assuming initially an arbitrary location of separation. The distribution of the velocity on the wall of the cylinder provided the outer flow velocity $U_e(x, t)$. The boundary layer equations were then solved to obtain a corrected position of separation and the process was repeated.

To facilitate comparison with other methods for the calculation of steady heat transfer over a circular cylinder, the Hiemenz outer flow distribution has been employed. This is a polynomial fitted to the experimental data (Spalding and Pun, 1962) given by the formula

$$U_o(x) = 3.631x - 3.275x^3 - 0.168x^5 \quad (27)$$

The coefficient a in Eq. 11 is therefore here equal to 3.631. The unsteady counterpart was given in the form

$$U_e(x, t) = U_o(x) + b \cos \omega t \quad (28)$$

which approximates closely the experimental data.

An array of cylinders in triangular configurations was also considered. In this case the steady part of the flow was calculated by viscous-inviscid interaction as described above. Unsteadiness was then assumed in the form of a uniform amplitude as in the case of the single cylinder. Two lines of symmetry were incorporated in the finite-element mesh. The model therefore is equivalent to two infinite cascades of cylinders and the heat transfer is studied on an element of the front row.

Results of the unsteady velocity field have been reported elsewhere by the present team (Telionis, 1981) and other investigators (Cebeci, 1979). Heat transfer rates have now been calculated first for the steady flow over a circular cylinder. A large number of such calculations have been reported in the literature but to the knowledge of the present authors, no one has attempted yet to solve the problem by a finite-difference method. This of course is today a fairly simple exercise, but it should be more reliable than any asymptotic method. Results of our calculations are shown in Figure 5 together with a number of other predictions. The references marked in the figure can be found in the review article of Spalding and Pun (1962). It is surprising that the experimental data considered as most reliable (Schmidt and Wenner, 1941) date back to the early 1940's. The scatter of analytical predictions is considerable, but the present results appear to be closest to the most respected predictions (Spalding and Pun, 1962).

It has been well known, and is clearly indicated in Figure 4, that all steady calculations underestimate experimental data of

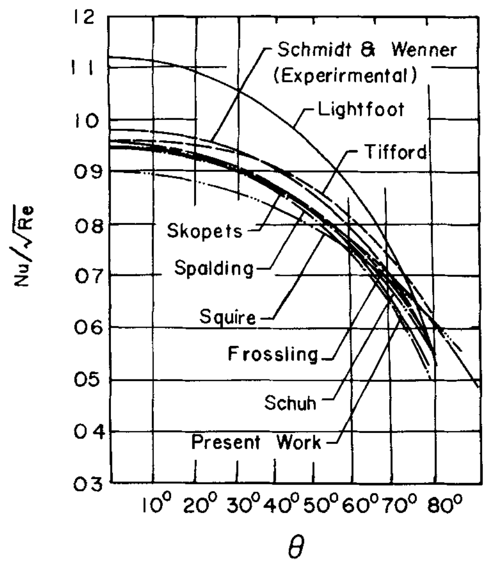

Figure 5. Heat transfer rates over attached flow region of a circular cylinder for steady oncoming flow. References are listed in Spalding and Pun (1962).

heat transfer. However, it has not been noticed that all experiments involve the natural unsteadiness of vortex shedding, which may introduce an amplitude of the outer flow as large as 10% of the mean stream. In other words, all theoreticians so far have compared their calculations for steady flow to experimental results that represent the time average of a naturally unsteady field. Only calculations of the unsteady field could predict correctly the phenomenon, and this has been one of the purposes of the present paper.

Results and Discussion

The unsteady boundary layer over a circular cylinder was calculated first by Dwyer and McCroskey (1973). They expressed their equations in terms of a moving reference frame attached to the oscillating stagnation point. The present calculations proceed in an alternative way. We employ the perturbation solution given in the Stagnation Flow section above as an initial condition, which then permits integration with respect to a fixed frame. Figure 6 displays a selection of temperature profiles within a period of the oscillation. It is emphasized again that both the free stream and the cylinder wall have fixed temperatures and therefore the temperature fluctuations are due to the nonlinear convection.

Averaged heat transfer rates over the attached region of the circular cylinder are shown in Figure 7 for a reduced frequency equal to the Strouhal frequency, namely 0.21 and a velocity amplitude $b = 0.1$. In the same figure the experimental data of Schmidt and Wenner (1941) as well as results of steady calculations are repeated from Figure 5 for comparison. The present results represent an improvement over earlier steady calculations. It is believed that the small discrepancy may be attributed to free stream turbulence, which Schmidt and Wenner have not reported.

Finally, in Figure 8 we show calculations of average heat transfer for three different values of the amplitude of oscillation. Our calculations indicate an increase in the heat transfer over the forward portion of the attached flow region, but a decrease of heat transfer in the neighborhood of separation.

For the bundle configuration examined here the results are presented in Figures 9 and 10. In Figure 9 we display the outer flow distributions of the velocity based on experimental data. It is seen in this figure that for a single cylinder our interacted solution approximates with a maximum error of 3% the experimental fit represented by the Hiemenz (Saxena and Laird, 1978) and Sogin (Sogin and Subramanian, 1961) curves. In the same figure the bundle solution displays much larger values of the velocity, which are attributed to the blockage effect. Finally, Figure 10 shows the heat transfer calculated with the outer flow distributions of Figure 9. Apparently the blockage generated by the proximity of the other cylinders induces larger velocities and heat transfers over the attached region of the cylinder.

Conclusions

The present study demonstrates how the condition of constant-pressure shear layer (free streamline) allows a finite-ele-

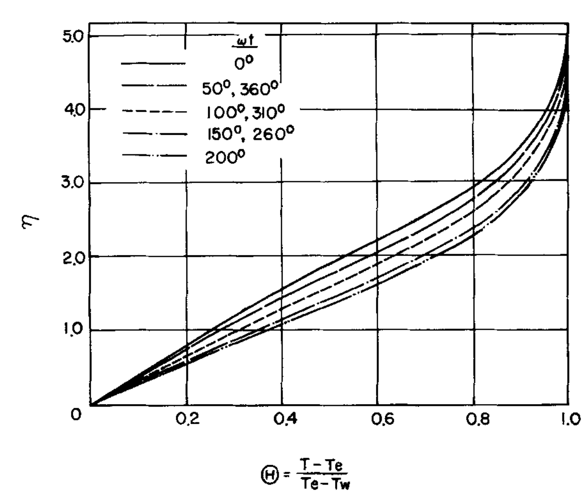

Figure 6. Temperature profiles for one period of oscillation, at a reduced frequency of 0.21.

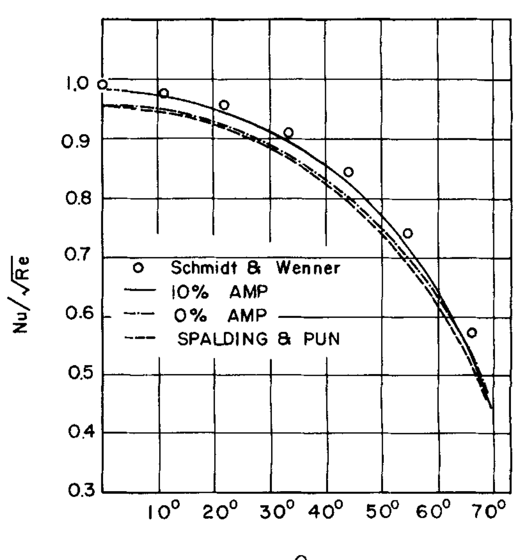

Figure 7. Mean heat transfer over a single circular cylinder.

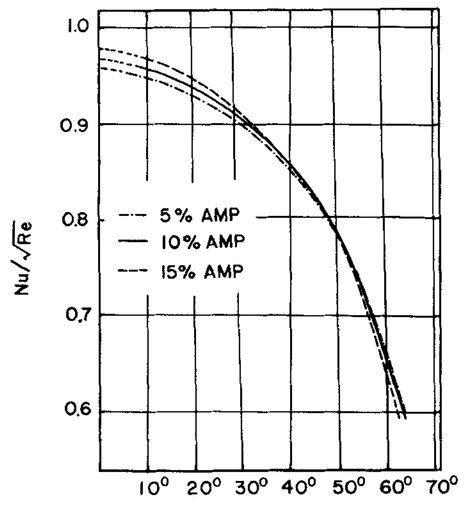

Figure 8. Averaged heat transfer over a circular cylinder for different values of the amplitude at a reduced frequency of 0.25.

ment inviscid method to predict the shape and size of separated regions. The technique can be applied to single- or multiple-body configurations and is valid for very high Reynolds numbers, a domain that is very hard to reach with full Navier-Stokes calculations. For periodic oncoming flow it is found that the heat transfer increases modestly in the attached flow region. Increases in the stagnation flow region are followed by small decreases in the upstream neighborhood of separation. It appears that small-amplitude periodic oscillations in the oncoming stream may indeed result in some increase of the attached-flow

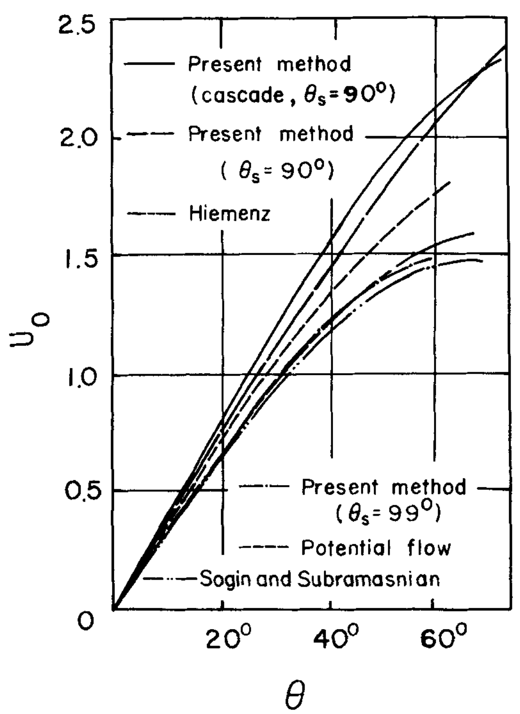

Figure 9. Outer flow velocity distribution.

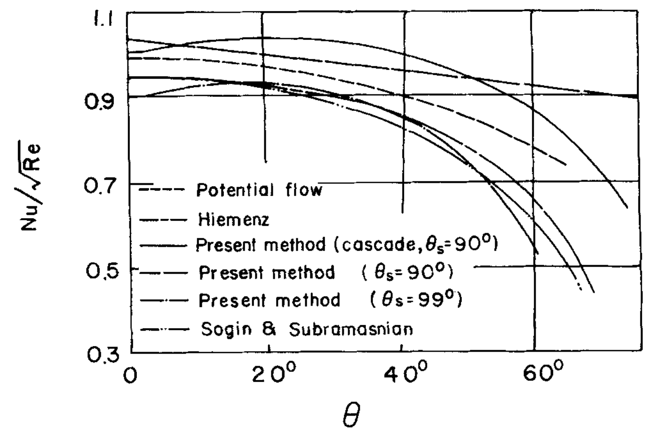

Figure 10. Steady heat transfer over a circular cylinder for various outer flow velocity distributions.

heat transfer. Recent experimental studies (Borell et al., 1984; Tolionis and Diller, 1985) indicate that heat transfer increases over the separated region as well. These findings may be taken into account in the design of modern heat exchangers. Externally imposed flow oscillations can indeed improve the overall efficiency of the system, without introducing any unnecessary mechanical vibrations. All such structures are designed with natural mechanical vibration frequencies away from natural shedding frequencies. The flow oscillations can thus be introduced safely at a frequency near the shedding frequency.

As a by-product of the present effort we found that natural shedding of vorticity also induces small increases of the heat transfer in the attached-flow region. This finding may be academic in nature but indicates why all earlier analytical predictions of heat transfer underestimated the experimental data.

The proposed method is applicable for very large Reynolds numbers and closed wakes of limited extent. This is because free-streamline theories predict with reasonable accuracy the mean boundaries of a fluctuating wake but fail to capture the details of large-scale vortex shedding. Moreover, in such cases the heat transfer is strongly affected by the mechanisms of violent mixing. However, a large number of practical engineering configurations involve tightly packed cylinders which dictate stable and confined separated regions. The present method is applicable for such configurations and is based on the assumption that the wakes are unbroken and stable. Heat transfer can be calculated in the wake by assuming a coherent recirculating flow pattern. This contribution is very small because such regions are limited in extent, and moreover, heat is transferred there mainly by diffusion.

The limitations on the geometrical configuration and the Reynolds numbers are severe, but on the other hand, the validity of the present method improves with increasing Reynolds numbers. It is well known that with increasing Reynolds numbers, the convergence of full Navier-Stokes equation solvers becomes more and more difficult. The present approximations therefore are more realistic and the solutions more reliable in a domain where it becomes more difficult to achieve convergence with classical methods.

Acknowledgment

The support of the U. S. Department of Energy, Grant No. DE-A505-82ER12022 is gratefully acknowledged. This is a project in the program of O. P. Manley, Division of Engineering, Mathematical and Geosciences of the Office of Basic Energy Sciences.

Notations

a = velocity gradient of stagnating flow
b = velocity amplitude
ds = surface differential element
$d\ell$ = linear differential element
Ec = Eckert number
$H(t)$ = Heaviside function
Pr = Prandtl number
Re = Reynolds number
t = time
T = temperature
T_e = temperature at edge of boundary layer
T_w = wall temperature
T_∞ = temperature at infinity
u = velocity component in x direction
U_e = boundary layer edge velocity
U_o = time mean of the edge velocity
u_n = velocity component normal to a surface
v = velocity component in y direction
x = cartesian coordinate aligned with body surface
y = cartesian coordinate normal to body surface

Greek letters

Δ = thickness equal to largest of displacement or momentum thickness
η = stretched boundary layer coordinate $\eta = U_e/\sqrt{2\xi}$
Θ = dimensionless temperature, $\Theta = (T - T_e)/(T_e - T_W)$
θ = azimuthal angle measured along the cylinder from the stagnation point, Figure 1
ϕ = velocity potential
ξ = stretched boundary layer coordinate, $\xi = \int_o^x U_e dx$
ω = angular velocity

Literature cited

Borell, G. J., B. K. Kim, W. Ekhaml, T. E. Diller, and D. P. Telionis, "Pressure and Heat Transfer Measurements Around a Cylinder in Pulsating Crossflow," 1984 ASME Fluids Eng. Conf., New Orleans (Feb., 1984).
Campbell, D. S., and T. E. Diller, "Design and Calibration of a Local Heat Flux Measurement System for Unsteady Flows," *Fundamentals of Forced and Mixed Convection,* F. A. Kulacki and R. D. Boyd, eds. (1985).
Cebeci, T., "The Laminar Boundary Layer on a Circular Cylinder Started Impulsively from Rest," *J. Comput. Phys.,* **31,** 153 (1979).
Dwyer, H. A., and W. J. McCroskey, "Oscillating Flow Over a Cylinder at Large Reynolds Number," *J. Fluid Mech.,* **61,** 753 (1973).
Eckert, E. R. G., and R. M. Drake, Jr., *Analysis of Heat and Mass Transfer,* McGraw-Hill, New York (1972).
Hribar, A. E., "Acoustic Streaming Near a Cylinder in Cross Flow," *J. Acoust. Soc.,* **51**(1), 95 (1972).
Kim, B. K., "Boundary Layer Analysis and Measurement of Newtonian and Non-Newtonian Fluids," Ph.D. Diss., Va. Polytech. Inst. State Univ. (Dec., 1984).
Kiya, Masaru, and Mikio Arie, "An Inviscid Bluff-Body Wake Model which Includes the Far-Wake Displacement Effect," *J. Fluid Mech.,* **81,** 593 (1977).
Lowery, C. W., and R. I. Vachon, "The Effect of Turbulence on Heat Transfer from Heated Cylinders," *Int. J. Heat Mass Transfer,* (18), 1229 (1975).
Martin, H. C., and G. F. Carey, *Introduction to Finite Element Analysis,* McGraw-Hill, New York (1973).
McCroskey, W. J., "Some Current Research in Unsteady Fluid Dynamics—The 1976 Freeman Scholar Lecture," *J. Fluids Eng., Trans. ASME,* **99,** 8 (1977).
Morgan, Vincent T., "The Overall Convective Heat Transfer from Smooth Circular Cylinders," *Adv. Heat Transfer,* J. P. Hartnett and T. F. Irvine, Jr., eds., Academic Press, New York, **11,** 199 (1975).
Repin, V. B., "Critical Sound Pressure Level in Mass Transfer Processes," *Sov. Physi. Acoust.,* **24**(2), 139 (1978).
Sarpkaya, T., "Vortex-Induced Oscillations—A Selective Review," *J. Appl. Mech.,* **46,** 241 (1979).
Savkar, S. D., "A Survey of Flow-Induced Vibrations of Cylindrical Arrays in Cross Flow," ASME Paper 76-WA/FE-21; *J. Fluids Eng.,* **99**(3), 517 (1976).
Saxena, V. C., and A. D. K. Laird, "Heat Transfer from a Cylinder Oscillating in a Cross Flow," *J. Heat Transfer, Trans. ASME,* **100,** 684 (1978).
Schmidt, E., and K. Wenner, "Heat Transfer Over the Circumference of a Heated Cylinder in Transverse Flow," NACA TM-1050 (1941).
Sears, W. R., and D. P. Telionis, "Boundary Layer Separation in Unsteady Flow," *SIAM J. Appl. Mech.,* **28,** 215 (1975).
Sogin, H. H., and V. S. Subramanian, "Local Mass Transfer from Circular Cylinders in Cross Flow," *J. Heat Transfer,* **2,** 15 (1961).
Spalding, D. B., and W. M. Pun, "A Review of Methods for Predicting Heat Transfer Coefficients for Laminar Uniform-Property Boundary Layer Flows," *Int. J. Heat Mass Transfer,* **5,** 239 (1962).
Sreenivasan, K., and A. Ramachandran, "Effect of Vibration on Heat Transfer from a Horizontal Cylinder to a Normal Air Stream," *Int. J. Heat Mass Transfer,* **3,** 60 (1961).
Strouhal, V., "Über eine Besondere Art der Tonerregung," *Anal. Physik Chemie,* **5,** 216 (1878).
Sunden, B., "A Theoretical Investigation of the Effect of Free-Stream Turbulence on Skin Friction and Heat Transfer for a Bluff Body," *Int. J. Heat Mass Transfer,* **22,** 1125 (1979).
Telionis, D. P., *Unsteady Viscous Flow,* Springer, New York, Heidelberg (1981).
Telionis, D. P., and T. E. Diller, "Periodic Flow and Heat Transfer Over Multiple Cylinders," 3rd Symp. Energy Eng. Sci., Penn. State Univ. (Oct., 1985).
Telionis, D. P., and M. S. Romaniuk, "Velocity and Temperature Streaming in Oscillating Boundary Layers," *AIAA J.,* **15,** 567 (1977).
Telionis, D. P., and D. T. Tsahalis, "Unsteady Laminar Separation Over a Cylinder Started Impulsively from Rest," *Acta Astronautica,* **1,** 1487 (1974).
Telionis, D. P., D. T. Tsahalis, and M. J. Werle, "Numerical Investigation of Unsteady Boundary Layer Separation," *Phys. Fluids,* **16,** 968 (1973).
Yardi, N. R., and S. P. Sukhatman, "Effects of Turbulence Intensity and Integral Length Scale of a Turbulent Free Stream on Forced Convection Heat Transfer from a Circular Cylinder in Cross Flow," in *Proc. 6th Int. Heat Transfer Conf., Toronto,* Hemisphere, Washington, DC, **5,** 347 (1978).
Zukauskas, A., "Heat Transfer from Tubes in Cross Flow," *Advances in Heat Transfer,* J. P. Hartnett and T. F. Irvine, Jr., eds., Academic Press, New York, **8,** 93 (1972).

Manuscript received Feb. 11, 1986, and revision received May 9, 1986.